Quantum state engineering with linear optical tools

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Introduction

Generation of special quantum states of light is still under intensive research in quantum optics. An efficient tool of quantum state engineering is based on discrete coherent-state superpositions. It has been shown that superposition of even a small number of coherent states put along a straight line or on a circle in phase space can approximate nonclassical field states with a high degree of accuracy [1, 2, 3]. In this communication we show that in the experimental scheme containing only beam splitters and homodyne detectors, discrete coherent state superpositions on a line and on a lattice in phase space can be produced with a certain degree of freedom in the coefficients. The states are prepared conditionally depending on the measurements result of the homodyne detectors in the scheme.
We have developed a numerical method for determining the parameters of homodyne measurements yielding a proper set of coefficients in the coherent state superposition with high fidelity. We demonstrate that squeezed coherent, photon number, and squeezed photon number states can be approximately prepared in the proposed scheme.

Discrete coherent-state superpositions on a line or a lattice in phase space

A nonclassical quantum state $|\psi_t\rangle$ can be approximated with high fidelity by discrete coherent-state superpositions (CSS) which consists only a few number of coherent states on a line or on a lattice in phase space with real parameter d. The fidelity between the target and the approximating state is

$$F_1 = |\left\langle \psi_{\mathrm{t}} | \psi_{\mathrm{app}} \right\rangle|, \tag{1}$$

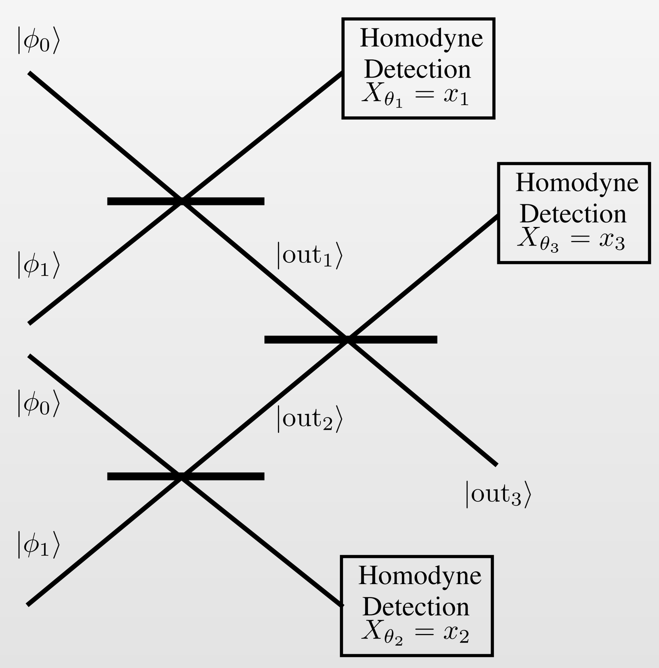
where the approximating state is a superposition of coherent states

$$|\psi_{\mathrm{app}}\rangle = \sum_{i=1}^{N} c_i |\alpha_i\rangle, \quad \sum_{i=1}^{N} |c_i|^2, \tag{2}$$

where the c_is are complex coefficients and $\alpha_i = \alpha_i(d)$.
Our aim is to find the optimal coefficients c_is and the parameter d which maximize the fidelity F_1.

Experimental scheme I.

The experimental scheme shown in the figure below only consists of linear optical elements: beam splitters and homodyne detectors. The state $|\mathrm{out}\rangle_3$ is prepared conditionally in travelling wave way. Recently, it has been shown that one element of this scheme can be used for preparing Schödinger-cat states [4, 5, 6].



The effect of a beam splitter on coherent states can be described by the following formula

$$\mathbf{BS}\Big\{ |\alpha\rangle_1 |\beta\rangle_2 \Big\} \rightarrow \left|\frac{\alpha+\beta}{\sqrt{2}}\right\rangle_3 \left|\frac{\alpha-\beta}{\sqrt{2}}\right\rangle_4 . \tag{3}$$

We perform a homodyne measurement on one of the modes of the two-mode light leaving the beam splitter, which eliminates the measured mode and introduces the following factor in the other mode

$$\langle x_\lambda | \alpha \rangle = \pi^{-\frac{1}{4}} \cdot \exp\left[\mathrm{i}\left\langle \hat{\mathbf{x}}_{\lambda+\frac{\pi}{2}}\right\rangle x_\lambda\right] \exp\left[-\frac{1}{2}(x_\lambda - \langle\hat{\mathbf{x}}_\lambda\rangle)^2\right] \exp\left[-\frac{1}{2}\mathrm{i}\langle\hat{\mathbf{x}}_\lambda\rangle \left\langle \hat{\mathbf{x}}_{\lambda+\frac{\pi}{2}}\right\rangle\right]. \tag{4}$$

The initial states for producing CSS on a line

$$|\phi_0\rangle = \left|\alpha e^{\mathrm{i}\frac{\pi+\varphi}{2}}\right\rangle + \left|\alpha e^{\mathrm{i}\frac{\pi-\varphi}{2}}\right\rangle, \qquad |\phi_1\rangle = \left|\alpha e^{\mathrm{i}\frac{\pi+\varphi}{2}}\right\rangle + \left|\alpha e^{\mathrm{i}\frac{\pi-\varphi}{2}}\right\rangle, \tag{5}$$

The initial states for producing CSS on a lattice

$$|\phi_0\rangle = \left|\alpha e^{\mathrm{i}\frac{\pi+\varphi}{2}}\right\rangle + \left|\alpha e^{\mathrm{i}\frac{\pi-\varphi}{2}}\right\rangle, \qquad |\phi_1\rangle = \left|\mathrm{i}\alpha e^{\mathrm{i}\frac{\pi+\varphi}{2}}\right\rangle + \left|i\alpha e^{\mathrm{i}\frac{\pi-\varphi}{2}}\right\rangle. \tag{6}$$

Experimental scheme II.

The effect of the beam splitters (3) and the homodyne measurements (4) on the initial states (5) and (6) are

$$|\mathrm{out}_3\rangle_{\mathrm{line}} = c_1|0\rangle + c_2|-d\rangle + c_3|d\rangle + c_4|2d\rangle + c_5|-2d\rangle,$$

$$|\mathrm{out}_3\rangle_{\mathrm{lattice}} = c_1|-d+\mathrm{i}d\rangle + c_2|\mathrm{i}d\rangle + c_3|d+\mathrm{i}d\rangle + c_4|-d\rangle + c_5|0\rangle + c_6|d\rangle + c_7|-d-\mathrm{i}d\rangle + c_8|-\mathrm{i}d\rangle + c_9|d-\mathrm{i}d\rangle,$$

where the c_i coefficients for the line are

$$\begin{aligned}
c_1 &= a_1 b_1 \langle X = x_3 | 0\rangle + a_2 b_2 \langle X = x_3 | 2d\rangle + a_2 b_2 \langle X = x_3 | -2d\rangle, \\
c_2 &= a_1 b_2 \langle X = x_3 | d\rangle + a_2 b_1 \langle X = x_3 | -d\rangle, \\
c_3 &= a_1 b_2 \langle X = x_3 | -d\rangle + a_2 b_1 \langle X = x_3 | d\rangle, \\
c_4 &= a_2 b_2 \langle X = x_3 | 0\rangle, \\
c_5 &= a_2 b_2 \langle X = x_3 | 0\rangle,
\end{aligned}$$

where $d = \frac{\alpha \cdot \varphi}{2}$ and the a_i and b_i coefficients are

$$a_1 = \left\langle X = x_1 \middle| \sqrt{2}\alpha \mathrm{i} e^{\mathrm{i}\frac{\varphi}{2}}\right\rangle_A + \left\langle X = x_1 \middle| \sqrt{2}\alpha \mathrm{i} e^{-\mathrm{i}\frac{\varphi}{2}}\right\rangle_A, \qquad a_2 = \left\langle X = x_1 \middle| \sqrt{2}\alpha \mathrm{i}\cos\frac{\varphi}{2}\right\rangle_A,$$

$$b_1 = \left\langle X = x_2 \middle| \sqrt{2}\alpha \mathrm{i} e^{\mathrm{i}\frac{\varphi}{2}}\right\rangle_A + \left\langle X = x_2 \middle| \sqrt{2}\alpha \mathrm{i} e^{-\mathrm{i}\frac{\varphi}{2}}\right\rangle_A, \qquad b_2 = \left\langle X = x_2 \middle| \sqrt{2}\alpha \mathrm{i}\cos\frac{\varphi}{2}\right\rangle_A,$$

where A denotes the measured mode and the parameter d is the same as in the previous approximation step. The fidelity between this output state and the approximating state in (2) is

$$F_2 = |\langle \psi_{\mathrm{app}} | \mathrm{out}_3\rangle|. \tag{7}$$

The optimal parameters x_1, x_2, x_3 maximizing the fidelity F_2 can be found numerically.

Results

- Squeezed coherent state $|\alpha = 1\rangle$ with squeezing parameter $\zeta = \frac{\mathrm{i}\pi}{12}$.
  The optimal parameters of the coherent-state superposition on a line:

c_1	c_2	c_3	c_4	c_5	d
0.1147 - 0.1101i	-0.0152 + 0.0748i	-0.6532 + 0.6535i	0.3082 + 0.1392i	-0.0008 - 0.0281i	0.88

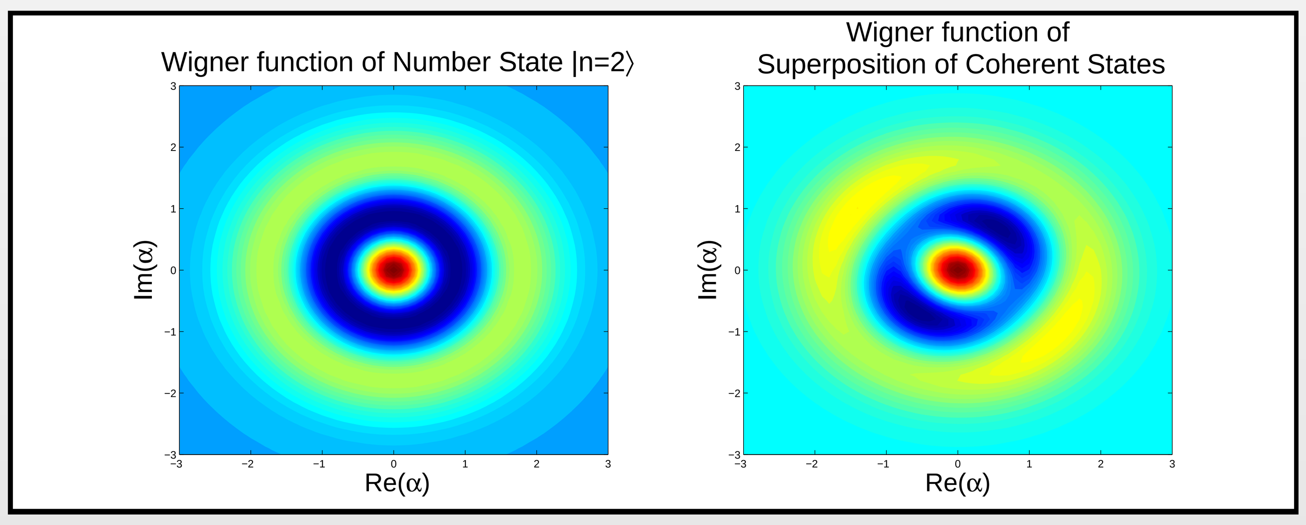
The optimal measurement parameters:

x_1	x_2	x_3
-0.169	-1.879	0.791

The fidelity of the target and the approximating state:

$$F = |\left\langle \psi_{\mathrm{t}} | \psi_{\mathrm{a}}\right\rangle| = 0.965. \tag{8}$$

- Approximating photon number state $|n = 2\rangle$ using discrete coherent-state superposition on a lattice:



The optimal parameters of the coherent-state superposition on a lattice:

c_1	c_2	c_3	c_4	c_5
-0.1334 - 0.3310i	0.1746 - 0.2665i	0.0771 + 0.1911i	0.0578 - 0.0108i	-0.0175 + 0.0105i
c_6	c_7	c_8	c_9	d
-0.5990 + 0.1835i	0.0863 + 0.3684i	0.4168 - 0.1015i	-0.0659 - 0.0297i	0.399

The optimal measurement parameters:

x_1	x_2	x_3
-0.499	38.78	-0.003

The fidelity of the target and the approximating state:

$$F = |\left\langle \psi_{\mathrm{t}} | \psi_{\mathrm{a}}\right\rangle| = 0.983. \tag{9}$$

- Squeezed photon number state $|n = 2\rangle$ with squeezing parameter $\zeta = \frac{\mathrm{i}\pi}{12}$.
  The optimal parameters of the coherent-state superposition on a lattice:

c_1	c_2	c_3	c_4	c_5
-0.5056 + 0.1441i	0.0471 - 0.2596i	-0.0218 - 0.0264i	0.2732 - 0.1533i	-0.0001 + 0.0018i
c_6	c_7	c_8	c_9	d
0.1011 + 0.1097i	0.1892 - 0.0959i	0.3445 - 0.0038i	-0.5188 + 0.3154i	0.96

The optimal measurement parameters:

x_1	x_2	x_3
-0.747	-1.026	0.002

The fidelity of the target and the approximating state:

$$F = |\left\langle \psi_{\mathrm{t}} | \psi_{\mathrm{a}}\right\rangle| = 0.928. \tag{10}$$

In summary, experimental scheme consisting only linear optical elements has been developed for producing conditionally nonclassical states of light in travelling wave optics.

References

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