Conditional generation of superpositions of photon number states of traveling fields

Gabor Mogyorosi[1], Emese Molnar[1], and Peter Adam[1,2]

[1]Institute of Physics, University of Pécs, H-7624 Pécs, Ifjúság útja 6, Hungary

[2]Institute for Solid State Physics and Optics,
Wigner Research Centre for Physics, HAS, H-1525 Budapest, P.O. Box 49, Hungary

Introduction

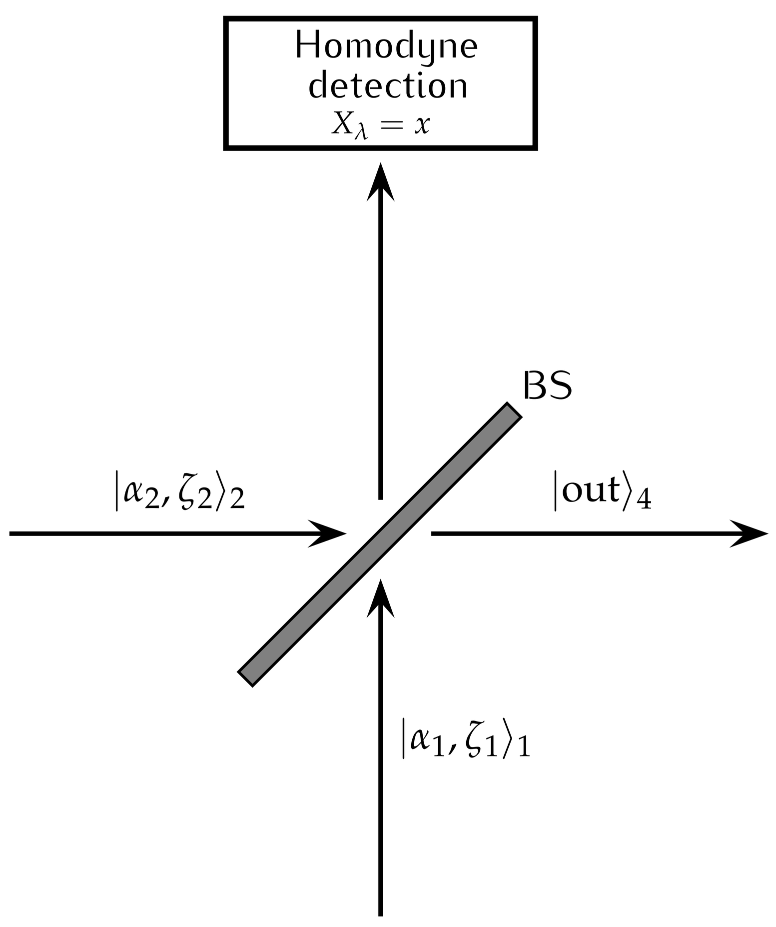
The problem of generating various quantum states of light is still an important topic in quantum optics, owing to their numerous applications in quantum information processing, quantum-enhanced metrology, and fundamental tests of quantum mechanics [1–11]. In this communication we show an experimental scheme, containing only a beam splitter of transmittance T and a homodyne detector capable of measuring the quadrature X_λ, it is possible to prepare various superpositions of photon number states, albeit with limited number of photons. The inputs of the scheme are independently prepared squeezed coherent states. The benefit of such input states is that they can be routinely generated experimentally by standard techniques.

A prescribed photon number superposition can be prepared on condition that a given measurement result $X_\lambda = x$ of the homodyne detector is obtained and the appropriate choice of the parameters α_i, ϕ_i, r_i, θ_i of the input states and the transmittance T of the beam splitter.

The required parameters can be determined numerically using a genetic algorithm. The objective is that the misfit between the target state and the output state should be minimal while the probability of conditional generation should be maximal. We demonstrate that the various superpositions of photon number states of small numbers, binomial and negative binomial states can be approximately prepared in the proposed scheme at a high accuracy and with large probability.

Experimental setup

Scheme for generating nonclassical states:



Beam splitter transformation:

$$\begin{pmatrix} \hat{a}_3 \\ \hat{a}_4 \end{pmatrix} = \begin{pmatrix} \sqrt{T} e^{i\phi_T} & \sqrt{1-T} e^{i\phi_R} \\ -\sqrt{1-T} e^{-i\phi_R} & \sqrt{T} e^{-i\phi_T} \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}, \tag{1}$$

where T is the transmittance and $\phi_T = 0$, $\phi_R = \dfrac{\pi}{2}$ are the phase angles of beam splitter.

Homodyne detection:

$$|X_\lambda = x\rangle\langle X_\lambda = x| \mathrm{d}x, \tag{2}$$

where

$$|X_\lambda = x\rangle = \frac{e^{-\frac{1}{2}x^2}}{\sqrt[4]{\pi}} \cdot \sum_{n=0}^{\infty} \frac{H_n(x) \cdot e^{in\lambda}}{\sqrt{2^n \cdot n!}} |n\rangle. \tag{3}$$

Inputs: The input states are single-mode squeezed coherent states:

$$|\alpha_i, \zeta_i\rangle = \frac{e^{-\frac{1}{2}|\alpha_i|^2 - \frac{1}{2}(\alpha_i^*)^2 e^{i\theta_i} \tanh(r_i)}}{\sqrt{\cosh(r_i)}} \cdot \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2} e^{i\theta_i} \tanh(r_i)\right)^{\frac{n}{2}}}{\sqrt{n!}} H_n\left(\beta_i \cdot \left[e^{i\theta_i} \sinh(2r_i)\right]^{-\frac{1}{2}}\right) |n\rangle, \tag{4}$$

where $\alpha_i = |\alpha_i| \cdot e^{i\phi_i}$, $\zeta_i = r_i \cdot e^{i\theta_i}$ and $\beta_i = \alpha_i \cosh(r_i) + \alpha_i^* e^{i\theta_i} \sinh(r_i)$. $\quad (i = 1, 2)$

Output state:

$$\begin{aligned} |\text{out}\rangle_4 = \mathcal{N}_{\text{out}} \cdot & \frac{e^{-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}(\alpha_1^*)^2 e^{i\theta_1} \tanh(r_1)} \cdot e^{-\frac{1}{2}|\alpha_2|^2 - \frac{1}{2}(\alpha_2^*)^2 e^{i\theta_2} \tanh(r_2)} \cdot e^{-\frac{1}{2}x^2}}{\sqrt[4]{\pi} \cdot \sqrt{\cosh(r_1) \cdot \cosh(r_2)}} \times \\ & \times \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{k=0}^{n} \sum_{l=0}^{m} \left(\frac{T}{4} e^{i(\theta_1 - 2\lambda)} \cdot \tanh(r_1)\right)^{\frac{n}{2}} \cdot \left(-\frac{(1-T)}{4} e^{i(\theta_2 - 2\lambda)} \cdot \tanh(r_2)\right)^{\frac{m}{2}} \times \\ & \times \binom{n}{k} \binom{m}{l} \cdot \frac{\sqrt{(k+l)!}}{n! \cdot m!} \cdot \left(\sqrt{2} e^{i\lambda}\right)^{k+l} \cdot \left(i\sqrt{\frac{1}{T} - 1}\right)^{k-l} \cdot H_{n-k+m-l}(x) \times \\ & \times H_n\left(\beta_1 \cdot \left[e^{i\theta_1} \cdot \sinh(2r_1)\right]^{-\frac{1}{2}}\right) \cdot H_m\left(\beta_2 \cdot \left[e^{i\theta_2} \cdot \sinh(2r_2)\right]^{-\frac{1}{2}}\right) |k+l\rangle_4. \end{aligned} \tag{5}$$

Numerical results

Numerical method:
Genetic algorithm for finding optimal parameters leading to minimal misfit:

$$\varepsilon = 1 - |\langle \text{out}|\Psi_{\text{target}}\rangle|^2, \tag{6}$$

where the quantity $|\langle \text{out}|\Psi_{\text{target}}\rangle|^2$ is the fidelity between the output and the target states.

Probability of the conditional generation, and the average misfit:

$$P(x^{\text{opt.}}, \delta) = \int_{x^{\text{opt.}} - \delta}^{x^{\text{opt.}} + \delta} \mathrm{Tr}\left(\hat{\varrho}_3 |x\rangle\langle x|\right) \mathrm{d}x, \quad \varepsilon_{\text{avg.}} = \frac{\sum_i \varepsilon_i \cdot P_i}{\sum_i P_i}, \tag{7}$$

where $\hat{\varrho}_3 = \mathrm{Tr}_4\left(|\text{out}\rangle_{34\,34}\langle \text{out}|\right)$, and δ is the measuring window.

Approximated nonclassical states:

- Arbitrary superpositions of photon number states:

$$|\Psi_{01}\rangle = \frac{1}{\sqrt{5}}(2|0\rangle + |1\rangle), \tag{8}$$

$$|\Psi_{012}\rangle = \frac{1}{3}(2|0\rangle + 2|1\rangle + |2\rangle), \tag{9}$$

$$|\Psi_{024}\rangle = \mathcal{N}_{\Psi_{024}}(|0\rangle + 0.29|2\rangle + 0.09|4\rangle), \tag{10}$$

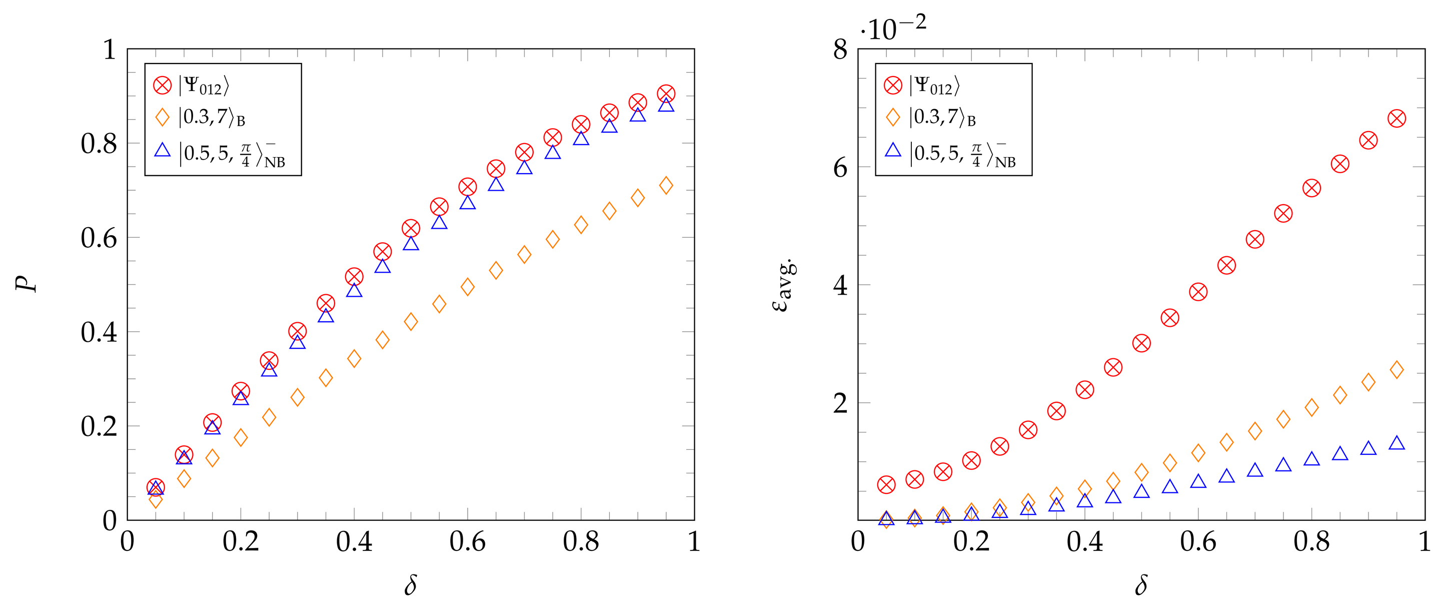
- Binomial and negative binomial states:

$$|p, M\rangle_{\text{B}} = \sum_{n=0}^{M} \left[\binom{M}{n} p^n (1-p)^{M-n}\right]^{\frac{1}{2}} |n\rangle, \tag{11}$$

$$|\eta, M, \varphi\rangle_{\text{NB}}^{-} = \sum_{n=0}^{\infty} \left[\binom{M+n-1}{n} \eta^{2n} (1-\eta^2)^M\right]^{\frac{1}{2}} e^{in\varphi} |n\rangle. \tag{12}$$

Results:

state	r_1	θ_1	α_1	ϕ_1	r_2	θ_2	α_2	ϕ_2	T	x	λ	ε	P	$\varepsilon_{\text{avg.}}$	δ
$\|\Psi_{01}\rangle$	0.18	1.02	0	0	0.22	0.28	0.36	0.08	0.7	0.23	0.64	6.4×10^{-4}	0.5	5.1×10^{-3}	0.45
$\|\Psi_{012}\rangle$	0.49	0.8	0.36	0.38	0.44	0.23	0.82	0.02	0.9	0.77	0.31	5.85×10^{-3}	0.46	1.86×10^{-2}	0.35
$\|\Psi_{024}\rangle$	0.91	0.38	0.11	0.46	0.06	0.82	0.1	0.39	0.7	0.27	0.37	1.35×10^{-3}	0.35	2×10^{-2}	0.25
$\|0.3, 7\rangle_{\text{B}}$	0.4	1.87	0.07	1.15	0.24	0.67	1.57	0.02	0.8	0.84	0.6	$1.13 \cdot 10^{-4}$	0.49	$6.7 \cdot 10^{-3}$	0.45
$\|0.6, 10\rangle_{\text{B}}$	0.18	1.97	0.32	1.01	0.68	0.11	2.8	0.02	0.8	1.07	0.31	4.85×10^{-3}	0.34	1.07×10^{-2}	0.4
$\|0.5, 5, \frac{\pi}{4}\rangle_{\text{NB}}^{-}$	0.39	0.82	0.85	0.54	0.25	0.04	1.58	0.45	0.65	0.88	0.28	3.36×10^{-5}	0.44	1.1×10^{-2}	0.4
$\|0.65, 1, 0\rangle_{\text{NB}}^{-}$	0.41	0.35	0.04	1.88	0.17	2.22	1.01	0.12	0.65	0.49	0.28	8.1×10^{-4}	0.43	6.9×10^{-3}	0.4



References

[1] A. Laghaout, J. S. Neergaard-Nielsen, I. Rigas, C. Kragh, A. Tipsmark, and U. L. Andersen, Phys. Rev. A **87**, 043826 (2013)

[2] K. Huang, H. Le Jeannic, V. B. Verma, M. D. Shaw, F. Marsili, S. W. Nam, E Wu, H. Zeng, O. Morin, and J. Laurat, Phys. Rev. A **93**, 013838 (2016)

[3] P. Adam, T. Kiss, M. Mechler, and Z. Darázs, Phys. Scr. **T140**, 014011 (2010)

[4] H. Jeong, M. S. Kim, T. C. Ralph, and B. S. Ham, Phys. Rev. A **70**, 061801(R) (2004)

[5] S. Szabo, P. Adam, J. Janszky, and P. Domokos, Phys. Rev. A. **53**, 2698 (1996)

[6] P. Adam, E. Molnar, G. Mogyorosi, A. Varga, M. Mechler, and J. Janszky, Phys. Scr. **90**, 074021 (2015)

[7] M. Dakna, J. Clausen, L. Knöll, and D.-G. Welsch, Phys. Rev. A **59**, 1658 (1999)

[8] J. Fiurášek, R. García-Patrón, and N. J. Cerf, Phys. Rev. A **72**, 033822 (2005)

[9] M. S. Kim, J. Phys. B **41**, 133001 (2008)

[10] S.-Y. Lee and H. Nha, Phys. Rev. A **82**, 053812 (2010)

[11] S. Wang, H.-C. Yuan, and X.-F. Xu, Eur. Phys. J. D **67**, 102 (2013)