Conditional generation of nonclassical states of traveling fields

Gabor Mogyorosi[1], **Emese Molnar**[1], **and Peter Adam**[1,2]

[1]Institute of Physics, University of Pécs, H-7624 Pécs, Ifjúság útja 6, Hungary

[2]Institute for Solid State Physics and Optics,
Wigner Research Centre for Physics, HAS, H-1525 Budapest, P.O. Box 49, Hungary

Introduction

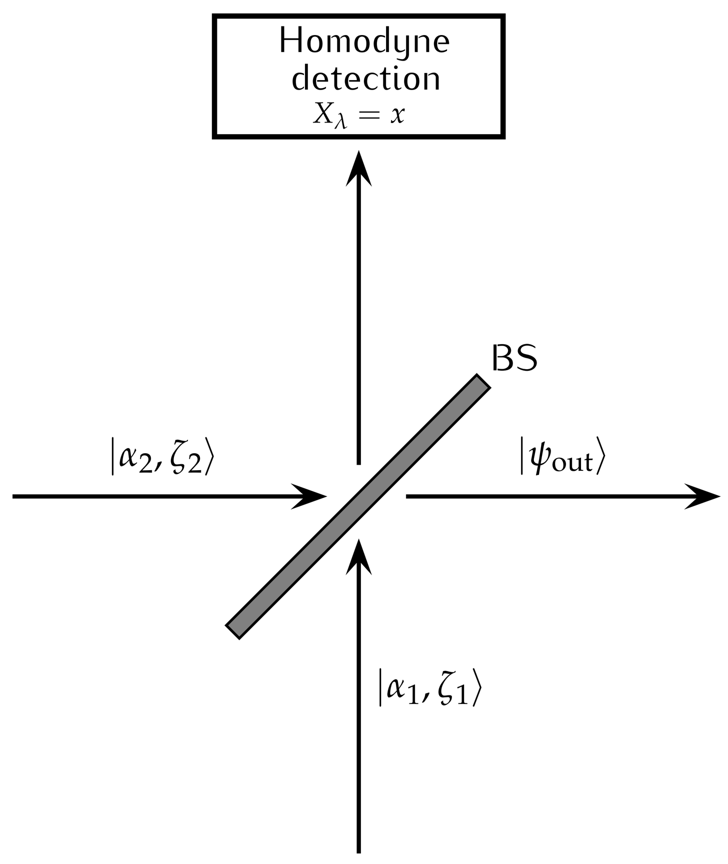
The problem of generating various quantum states of light is still an important topic in quantum optics, owing to their numerous applications in quantum information processing, quantum-enhanced metrology, and fundamental tests of quantum mechanics [1–6]. A considerable attention has been devoted also to the idea of quantum state engineering, i.e., to the preparation of several different nonclassical states in a single experimental scheme. The generation of nonclassical states of traveling optical modes is often desired in practical application. Conditional preparation is a well-established technique for such a task.

We show that in the experimental scheme, containing only a beam splitter of transmittance T and a homodyne detector capable of measuring the quadrature X_λ, it is possible to prepare various superpositions of photon number states, albeit with limited number of photons. The inputs of scheme are two independently prepared squeezed coherent states. The prescibed photon number superpositions corresponding to a given nonclassical state can be prepared with appropriate choice of the parameters of input states and the transmittance of beam splitter and on condition that a given measurement results of the homodyne detector is obtained.

The required parameters can be determined numerically using a genetic algorithm. The objective is that the misfit between the target state and the output state should be minimal while the probability of conditional generation should be maximal. In the proposed scheme various nonclassical states can be approximately prepared at a high accuracy and with large probability.

Experimental setup

Scheme for generating nonclassical states:



Beam splitter transformation:

$$\begin{pmatrix} \hat{a}_3 \\ \hat{a}_4 \end{pmatrix} = \begin{pmatrix} \sqrt{T}e^{\mathrm{i}\phi_T} & \sqrt{1-T}e^{\mathrm{i}\phi_R} \\ -\sqrt{1-T}e^{-\mathrm{i}\phi_R} & \sqrt{T}e^{-\mathrm{i}\phi_T} \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}, \tag{1}$$

where $\phi_T = 0$, $\phi_R = \pi/2$ are the phase angles of beam splitter.

Homodyne detection:

$$|X_\lambda = x\rangle\langle X_\lambda = x| \mathrm{d}x, \tag{2}$$

where

$$|X_\lambda = x\rangle = \frac{e^{-\frac{1}{2}x^2}}{\sqrt[4]{\pi}} \cdot \sum_{n=0}^{\infty} \frac{H_n(x) \cdot e^{\mathrm{i}n\lambda}}{\sqrt{2^n \cdot n!}} |n\rangle. \tag{3}$$

Inputs: The input states are single-mode squeezed coherent states:

$$|\alpha_i,\zeta_i\rangle = \frac{e^{-\frac{1}{2}|\alpha_i|^2 - \frac{1}{2}(\alpha_i^*)^2 e^{\mathrm{i}\theta_i}\tanh(r_i)}}{\sqrt{\cosh(r_i)}} \cdot \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}e^{\mathrm{i}\theta_i}\tanh(r_i)\right)^{\frac{n}{2}}}{\sqrt{n!}} H_n\left(\beta_i \cdot \left[e^{\mathrm{i}\theta_i}\sinh(2r_i)\right]^{-\frac{1}{2}}\right)|n\rangle, \tag{4}$$

where $\alpha_i = |\alpha_i| \cdot e^{\mathrm{i}\phi_i}$, $\zeta_i = r_i \cdot e^{\mathrm{i}\theta_i}$ and $\beta_i = \alpha_i \cosh(r_i) + \alpha_i^* e^{\mathrm{i}\theta_i}\sinh(r_i)$. $\qquad (i = 1, 2)$

Output state:

$$\begin{aligned} |\psi_{\text{out}}\rangle = {} & \mathcal{N}_{\text{out}} \cdot \pi^{-\frac{1}{4}} \cdot e^{-\frac{1}{2}x^2} \cdot \prod_{j=1}^{2} \frac{\exp\left(-\frac{1}{2}|\alpha_j|^2 - \frac{1}{2}\alpha_j^{*2}e^{\mathrm{i}\theta_j}\tanh(r_j)\right)}{\sqrt{\cosh(r_j)}} \times \\ & \times \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{k=0}^{n}\sum_{l=0}^{m}(-1)^l \left(\frac{1}{4}e^{\mathrm{i}(\theta_1-2\lambda)}\tanh(r_1)\right)^{\frac{n}{2}} \left(-\frac{1}{4}e^{\mathrm{i}(\theta_2-2\lambda)}\tanh(r_2)\right)^{\frac{m}{2}} \times \\ & \times \frac{\sqrt{(k+l)!}}{n! \cdot m!}\left(\sqrt{2}\mathrm{i}e^{\mathrm{i}\lambda}\right)^{k+l} \cdot B_k^n(\sqrt{T}) B_l^m(\sqrt{1-T}) \times \\ & \times H_{n+m-(k+l)}(x) \cdot H_n\left(\beta_1[e^{\mathrm{i}\theta_1}\sinh(2r_1)]^{-\frac{1}{2}}\right) H_m\left(\beta_2[e^{\mathrm{i}\theta_2}\sinh(2r_2)]^{-\frac{1}{2}}\right)|k+l\rangle. \end{aligned} \tag{5}$$

Numerical results

Numerical method:
Genetic algorithm for finding optimal parameters leading to minimal misfit:

$$\varepsilon = 1 - |\langle\psi_{\text{out}}|\Psi_{\text{target}}\rangle|^2, \tag{6}$$

where the quantity $|\langle\psi_{\text{out}}|\Psi_{\text{target}}\rangle|^2$ is the fidelity between the output and the target states.

Probability of the conditional generation, and the average misfit:

$$P(x^{\text{opt}},\delta) = \int_{x^{\text{opt}}-\delta}^{x^{\text{opt}}+\delta} \mathrm{Tr}\,(\hat{\varrho}_3|x\rangle\langle x|)\,\mathrm{d}x, \quad \varepsilon_{\text{avg.}} = \frac{\sum_i \varepsilon_i \cdot P_i}{\sum_i P_i}, \tag{7}$$

where $\hat{\varrho}_3 = \mathrm{Tr}_4\left(|\psi_{\text{out}}\rangle_{3434}\langle\psi_{\text{out}}|\right)$, and δ is the measuring window.

Approximated nonclassical states:

- Arbitrary superpositions of photon number states:

$$|\Psi_{01}\rangle = \frac{1}{\sqrt{5}}(2|0\rangle + |1\rangle), \tag{8}$$

$$|\Psi_{012}\rangle = \frac{1}{3}(2|0\rangle + 2|1\rangle + |2\rangle), \tag{9}$$

$$|\Psi_{024}\rangle = \mathcal{N}_{\Psi_{024}}(|0\rangle + 0.29|2\rangle + 0.09|4\rangle), \tag{10}$$

- Binomial, negative binomial and amplitude squeezed states:

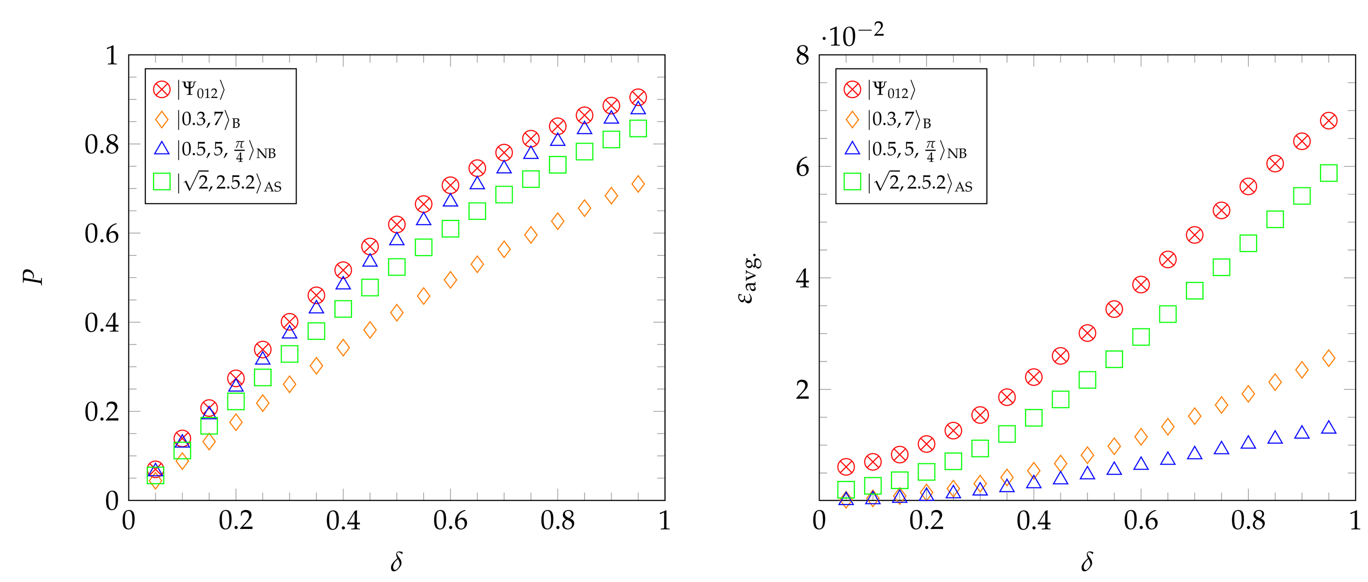
$$|p, M\rangle_{\text{B}} = \sum_{n=0}^{M}\left[\binom{M}{n}p^n(1-p)^{M-n}\right]^{\frac{1}{2}}|n\rangle, \tag{11}$$

$$|\eta, M, \varphi\rangle_{\text{NB}} = \sum_{n=0}^{\infty}\left[\binom{M+n-1}{n}\eta^{2n}(1-\eta^2)^M\right]^{\frac{1}{2}} e^{\mathrm{i}n\varphi}|n\rangle, \tag{12}$$

$$|\alpha_0, u, \delta\rangle_{\text{AS}} = \mathcal{N} \cdot \sum_{n=0}^{\infty}\frac{\sqrt{2\pi}\alpha_0^n}{u\sqrt{n!}}\exp\left[-\frac{(\delta-n)^2}{2u^2}\right]|n\rangle. \tag{13}$$

Results:

state	r_1	θ_1	α_1	ϕ_1	r_2	θ_2	α_2	ϕ_2	T	x	λ	ε	P	$\varepsilon_{\text{avg.}}$	δ
$\lvert\Psi_{01}\rangle$	0.18	1.02	0	0	0.22	0.28	0.36	0.08	0.7	0.23	0.64	6.4×10^{-4}	0.5	5.1×10^{-3}	0.45
$\lvert\Psi_{012}\rangle$	0.49	0.8	0.36	0.38	0.44	0.23	0.82	0.02	0.9	0.77	0.31	5.85×10^{-3}	0.46	1.86×10^{-2}	0.35
$\lvert\Psi_{024}\rangle$	0.91	0.38	0.11	0.46	0.06	0.82	0.1	0.39	0.7	0.27	0.37	1.35×10^{-3}	0.35	2×10^{-2}	0.25
$\lvert 0.3,7\rangle_{\text{B}}$	0.4	1.87	0.07	1.15	0.24	0.67	1.57	0.02	0.8	0.84	0.6	$1.13 \cdot 10^{-4}$	0.49	$6.7 \cdot 10^{-3}$	0.45
$\lvert 0.6,10\rangle_{\text{B}}$	0.18	1.97	0.32	1.01	0.68	0.11	2.8	0.02	0.8	1.07	0.31	4.85×10^{-3}	0.34	1.07×10^{-2}	0.4
$\lvert 0.5,5,\frac{\pi}{4}\rangle_{\text{NB}}$	0.39	0.82	0.85	0.54	0.25	0.04	1.58	0.45	0.65	0.88	0.28	3.36×10^{-5}	0.44	1.1×10^{-2}	0.4
$\lvert 0.65,1,0\rangle_{\text{NB}}$	0.41	0.35	0.04	1.88	0.17	2.22	1.01	0.12	0.65	0.49	0.28	8.1×10^{-4}	0.43	6.9×10^{-3}	0.4
$\lvert 1,2,1\rangle_{\text{AS}}$	0.37	1.61	1.29	2.4	0.23	0.86	1.78	0.36	0.7	1.71	3.1	1.22×10^{-3}	0.37	6.9×10^{-3}	0.4
$\lvert\sqrt{2},2.5,2\rangle_{\text{AS}}$	0.65	0.64	1.5	2.19	0.33	0.6	1.54	0.39	0.8	0.75	2.73	1.81×10^{-3}	0.22	5.2×10^{-3}	0.2



References

[1] S. Szabo, P. Adam, J. Janszky, and P. Domokos, Phys. Rev. A. **53**, 2698 (1996)

[2] H. Jeong, M. S. Kim, T. C. Ralph, and B. S. Ham, Phys. Rev. A **70**, 061801(R) (2004)

[3] P. Adam, T. Kiss, M. Mechler, and Z. Darázs, Phys. Scr. **T140**, 014011 (2010)

[4] A. Laghaout, J. S. Neergaard-Nielsen, I. Rigas, C. Kragh, A. Tipsmark, and U. L. Andersen, Phys. Rev. A **87**, 043826 (2013)

[5] P. Adam, E. Molnar, G. Mogyorosi, A. Varga, M. Mechler, and J. Janszky, Phys. Scr. **90**, 074021 (2015)

[6] K. Huang, H. Le Jeannic, V. B. Verma, M. D. Shaw, F. Marsili, S. W. Nam, E Wu, H. Zeng, O. Morin, and J. Laurat, Phys. Rev. A **93**, 013808 (2016)