Entanglement in quantum bit cloning and in Hardy's paradox

A summary of the PhD Thesis of

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I. The subject of the dissertation and the goals of the research

Quantum mechanics is the fundemental theory of all modern physics. As such it is a significant part of our understanding of Nature. It employs sophisticated mathematical models whose interpretation still poses unresolved physical and philosophical questions. Some of these lead to phenomena which are unusual and counterintuitive from an everyday perspective. In spite of this, quantum mechanics is a very successful theory. While its grounding fathers considered it as a theory for multipartite systems not verifyable on the level of individual physical systems, the formidable development of experimental technology (especially that of quantum optics) in the last decades has made the direct observation of these conterintuitive phenomena viable. And indeed: quantum mechanics appears to be valid for individual physical systems. Moreover, the accessibility of quantum phenomena seems to find its way to practical applications. The paradigm shift in physics introduced by quantum mechanics seems to be repeated in the field of information theory and information processing: quantum information now has a well-established reputation amongst future and emerging technologies. While quantum random generators and some quantum ciphers are commercial products already, yet there are still a lot of details to be better understood.

In my thesis I presented my results concerning with two main topics. One of them was the entanglement manipulation capabilities of the universal covariant quantum cloner or quantum processor circuit for quantum bits, and the other one was a similarly interesting theme where I analyzed Hardy's paradox from the point of view of scattering theory. My work belongs to the area of quantum information theory. The phenomenon of quantum entanglement plays a fundamental role in both of the mentioned topics.

Using certain quantifications of entanglement I have analyzed the entanglement manipulation capabilities of the universal covariant quantum cloner or quantum processor circuit for quantum bits. In the analysis the cloning a member of a bipartite or a genuine tripartite entangled state of quantum bits was considered. In case of bipartie states question to be answered was how much of bipartite entaglement "remains" between the original qubit and the other member of the pair, and how much of it is "transferred” to the clone when varying the cloning fidelity. I found that for bipartite pure entangled states a nontrivial behavior of concurrence appears. I have also studied the situation for cloning a member of GHZ entangled states, which are genuine threepartite states. The question here is the quantification of bipartie entanglements which are availabe via measurements on certain subsystems. I found that possibility of the partial extraction of bipartite entanglement can be achieved. These procedures can be useful in quantum communication and computation protocols.

The study of Hardy’s paradox from the point of view of scattering theory was motivated by the fact that this approach was useful for the understanding of interaction-free measurement, which is a similar setup. I have calculated the forward-scattered waves generated by the beam splitters, which are replaceable in the gedanken experiment. I pointed out that these two-mode waves appear to have an entanglement-like structure, reflecting the quantum nature of the phenomenon.

Since there is a photon interferometric scenario which is directly similar to that of the gedanken experiment of Hardy, where the annihilation of the particle- antiparticle pair is replaced by the interference of the two photons on a beam splitter, it was interesting to analyze its relation to Hardy's paradox. I calculated the forward-scattered waves of the output beam splitters for this setup and analyzed their entanglement-like structure. I found similarities with the original paradox.

II. New scientific results

My new scientific results are enumerated below. The list of scientific publications concerning these results can be found in the last point of this summary.

1. I have shown that when using a universal covariant quantum cloning circuit to clone a member of an entangled pair of qubits, when a very specific behavior of the entanglement of the qubits appears in its dependence on the cloning fidelity. The main feature is that behavior of the entanglement between the not cloned part of the pair and the cloned one is repeated in the entanglement of certain ancillae, and so is that of the not cloned qubit and the clone, provided that the original qubit pair was maximally entangled initially. My thesis includes the detailed description of the behavior of

entanglement. Also in this investigation, I showed that analyzed the cloning of an element of the GHZ state. I have shown that the universal quantum cloning circuit facilitates the partial extraction of bipartite entangled resources from a genuine tripartite entangled resource. I gave a detailed analysis of the entanglement behavior, including the relation to Coffman-Kundu-Wootters inequalities.

In summary, I have shown that the universal quantum cloning circuit (or quantum processor) for qubits can be used as an entanglement manipulator as well. It can perform entanglement manipulations which are potentially applicable in quantum information processing [I].

2. I have studied the interferometric setup of Hardy's paradox from the point of view of the structure of the for-

ward-scattered waves when the beam splitters of the scheme are considered as scattering centers. I showed that in the original setup, the forward-scattered wave is not of a product structure, and the forward-scattered wave for both beam splitters is not simply a linear combination of that of the two beam splitters. The forward-scattered wave has zero amplitude for the photon possibly generated in the system, which illustrates the relation between Hardy's paradox and interaction-free measurement. This approach might prove to be a good tool for the analysis of such setups[II].

3. I have investigated a realizable experimental setup that is directly similar to the one that was dealt with in the previous point. In this setup the role of particle-antiparticle annihilation was played by the interference

of two photons on the beam splitter. I have calculated the respective forward-scattered waves. Though the values of the quantities featuring the entanglement were not equal to the values obtained in the case of Hardy's original gedanken experiment, the behaviour of the two setups showed similarities[III].

III. List of related publications

I. L. Szabó, M. Koniorczyk, P. Adam, and J. Janszky: *Optimal universal asymmetric covariant quantum cloning circuits for qubit entanglement manipulation* Phys. Rev. A **81**, 032323 (2010).

II. M. Koniorczyk, L. Szabó and P. Adam: *Hardy's paradox and the entanglementlike structure of forward-scattered waves*, Phys. Rev. A **84**, 044102 (2011).

III. P. Adam, L. Szabó, M. Mechler, M. Koniorczyk: *Forward-scattered wave analysis of an optical Hardy-like setup*, Phys. Scr. **T147**, 014001/1-4 (2012).